

Developing a Gaussian Model-Based Histogram Equalization Technique for Enhancement of Breast Thermography Images

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Abstract

Purpose: Breast cancer is one of the most prevalent diseases among women worldwide. One of the effective ways to reduce the risk of death from breast cancer is early detection by breast screening methods such as thermography. Thermography is non-invasive infrared imaging that detects early symptoms of breast angiogenesis based on the temperature difference and asymmetric patterns between left and right breasts. For better visual perception, it is essential to increase the medical image quality and contrast.

Materials and Methods: Histogram Equalization (HE) is a common and effective technique for contrast enhancement that uses the whole dynamic range of gray levels. In this paper, we propose to apply the equalization technique to the object part of the image rather than the background. One way is to use Otsu's method for automatic image thresholding. A more efficient approach to extract the body region is to fit a bimodal Gaussian distribution on the temperature information and restrict the equalization on gray level ranges corresponding to temperatures between the mean minus/plus three times of standard deviation.

Results: We compared the performance of the proposed approach with six conventional HE methods by using objective criteria, including Absolute Mean Brightness Error (AMBE), Peak Signal-to-Noise Ratio (PSNR), Structural Similarity Index (SSI), and Entropy.

Conclusion: Based on objective measures, as well as subjective visual inspection of the results, the proposed Gaussian model-based HE has better performance in contrast enhancement and brightness preservation among other methods.

Keywords: Breast Thermography; Histogram Equalization; Gaussian Model; Contrast Enhancement; Objective Criteria.

1. Introduction

Breast cancer is the most prevalent cancer among women which is ranked the second leading reason of cancer deaths [1]. Although the possibility of breast cancer is affected by age and genetic [2], it should be noted that breast screening and early detection are important factors in increasing awareness and consequently decreasing mortality [3]. Therefore, various methods are used for diagnosing breast cancer such as clinical breast exams, X-ray mammography, ultrasound, and Magnetic Resonance Imaging (MRI) screening. One of the medical imaging techniques that is used for breast abnormality detection is thermography. Since there is a high temperature and blood activity in tumor cells [4], this method differentiates the normal and tumorous regions by measuring the surface temperature based on infrared radiation [5]. Thermography is considered an efficient, non-invasive, and painless method in comparison to any other modality, which is able to detect prior symptoms much earlier. Also, there is not a limitation in breast density and age of the patient in this modality [6]. Early detection of abnormalities by specialists requires that the thermography images should have a high quality and contrast.

Histogram Equalization (HE) is a technique that provides better visual perception by contrast enhancement through uniform redistributing gray levels in the whole dynamic range of the image [7]. This technique has various versions including Global Histogram Equalization (GHE), Brightness preserving Bi-Histogram Equalization (BBHE), equal-area Dualistic Sub-Image Histogram Equalization (DSIHE), Local Histogram Equalization (LHE), Contrast Limited Adaptive Histogram Equalization (CLAHE), and Adaptive Gamma Correction with Weighted Distribution (AGCWD). These methods are thoroughly explained in the method section.

Medical images are used vastly in diagnosing different diseases and tumors; therefore, they must have high quality and contrast with as slight noise as possible. Studies have shown that HE is one of the common techniques to enhance the contrast and brightness of different medical images. As an example, CLAHE has been applied to MRI breast images prior to extraction of the tumorous region [8]. Also, Adaptive Histogram Equalization (AHE) has been used as a preprocessing step to increase the contrast of mammography images [9] as well as ultrasound breast images [10]. Therefore, different methods of HE are used to improve the contrast of medical images such as breast

images, increase their quality, and consequently, help the specialist to make certain decisions more clearly.

In this paper, we aim to propose a HE technique that considers the content of the medical images and is suitable for contrast enhancement of breast thermography images. This technique is based on fitting a bimodal Gaussian model to the image histogram. Gaussian mixture models have been shown a good method for automatic segmentation of these images, too [11]. Besides subjective and visual inspection of the results, we compare the proposed method with existing ones by objective criteria, including Absolute Mean Brightness Error (AMBE), Peak Signal-to-Noise Ratio (PSNR), Structure Similarity Index (SSI), and entropy. The complete description of the dataset is provided in the next section.

2. Materials and Methods

2.1. Dataset

The dataset consisted of 50 breast thermography images which were taken by Thermoteknix VisIR 640 camera in “Imam Hospital”, Tehran, Iran [11]. The thermal sensitivity of the camera was 50 mK. The camera contained an uncooled focal plane array detector of resolution 640×480 with detector pitch equal to $25 \mu\text{m}$. The spectral sensitivity of the detector was in the range of 7.5 to $13 \mu\text{m}$. According to Plank’s equation and Wein’s Law, it is found that approximately 90% of the emitted infrared radiation is in the range of 6 to $14 \mu\text{m}$ [12]; therefore the sensitivity of the used camera was acceptable. The field of view was $26^\circ \times 20^\circ$ and the camera was placed 1 m away from the patient’s chest. The temperature of the examination room was in the range of 20 - 22°C (within $\pm 0.1^\circ\text{C}$). Since stabilization and reduction of the basal metabolic rate is essential to minimize surface temperature changes [13], patients were asked to rest for at least 15 minutes. For each person, there is an excel file containing the temperature information as well as a gray-level image. Visual inspection of the two types of data shows a linear transformation. Figure 1 shows a sample of images, its histogram, temperature histogram, and the scatterplot of gray levels with respect to temperature values.

2.2. Histogram Equalization (HE)

Histogram is one of the important features of an image that describes the frequency of occurrence of the gray

levels in the image. By looking at an image histogram, the entire distribution of intensity, in addition to dynamic range, can be obtained [14].

Assume that $X = \{X(i, j)\}$ is an image in which $X(i, j)$ represents the intensity at spatial location (i, j) . For an image X with L discrete gray levels denoted by $\{X_0, X_1, \dots, X_{L-1}\}$, the histogram h is defined as (Equation 1):

$$h(X_k) = n_k, \text{ For } k = 0, 1, \dots, L - 1 \quad (1)$$

Where X_k is the k -th gray level and n_k is the number of pixels with the same intensity level k .

The histogram also can be defined by the Probability Density Function (PDF), and it can be obtained by normalizing the histogram [15]. For an image X , the PDF for intensity X_k is defined by (Equation 2):

$$p(X_k) = \frac{n_k}{N}, \text{ For } k = 0, 1, \dots, L - 1 \quad (2)$$

Where N is the total number of pixels in the image, and n_k is the number of pixels with the same intensity level k . Graphical representation of PDF is the histogram of the image.

HE, also known as GHE, is a well-known method that enhances the image contrast by redistributing the intensity over the full range of gray levels in a uniform way [16].

By calculating the sum of all components of PDF, Cumulative Density Function (CDF) is obtained which is given by the following Equation 3:

$$C(X_k) = \sum_{i=0}^k p(X_i), \text{ For } k = 0, 1, \dots, L - 1 \quad (3)$$

Where $C(X_k)$ is CDF, $C(X_k) = 1$. HE uses CDF as its transformation function, $T(x)$, which is defined as (Equation 4):

$$T(X_k) = (L - 1).C(X_k) \text{ For } k = 0, 1, \dots, L - 1 \quad (4)$$

Although HE increases the overall contrast of the image, it has also some limitations. It has a well-known problem, “mean-shift”, which causes a shift in the mean intensity and consequently, a difference between the mean brightness of input and output images. Therefore, it is not a suitable technique where the brightness preservation of the image is essential. By this method, the noise in the image is also enhanced which causes artifacts and unnatural enhancement. Thus, although HE is an effective technique in increasing the contrast and brightness of the image, it might decrease the quality of the image [15].

2.3. Brightness Preserving Bi Histogram Equalization (BBHE)

To overcome the mentioned mean-shift problem, Brightness preserving Bi Histogram Equalization (BBHE) was proposed [17]. In this method, in addition to contrast enhancement, the mean brightness of the image is preserved. The BBHE decomposes the input image into two sub-images with different intensity ranges, one from minimum gray level to mean value and the other one from

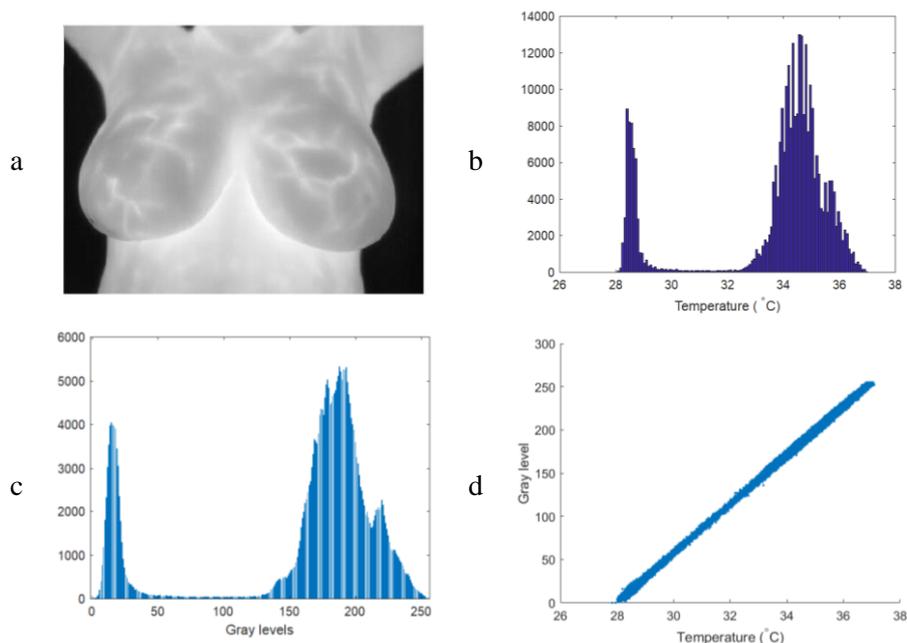


Figure 1. A sample of the available images (a), histogram of pixels' temperature (b), histogram of pixels' gray-level value (c), and the linear relationship between gray-level values and temperature (d)

mean value to maximum gray level. Then, it equalizes the histogram of each sub-image independently.

2.4. Equal Area Dualistic Sub-Image Histogram Equalization (DSIHE)

Equal area DSIHE has the same idea as the BBHE method and decomposes the input image into two sub-images; but decomposition is based on median value, instead of mean value as in BBHE [16]. The goal of this method is to maximize Shannon's entropy of the output image. After decomposing the input image based on gray levels with CDF value equal to 0.5, which leads to two dark and bright sub-images, the HE is applied on each sub-image. Composing equalized sub-images into one image results in DSIHE output image [14].

2.5. Local Histogram Equalization (LHE)

As it was mentioned in section 2.2, HE enhances the input image globally without considering its details. Since HE uses one CDF as a transformation function for all of the pixels, the local content of the image remains without enhancement [15]. By LHE, also known as AHE, this limitation is overcome.

In the LHE method, a window, W , of dimension $M \times M$ is defined and only the pixels of this window are used for the calculation of CDF. In this way, a local transformation function is defined for each pixel based on the neighborhood pixels. In other words, the histogram of the window is equalized and the transformation of the central pixel is calculated [18]. PDF of the window W is defined as (Equation 5):

$$p_W(X_k) = \frac{n_k}{M^2}, \text{ For } k = 0, 1, \dots, L - 1 \quad (5)$$

Where n_k is the number of pixels in the window with gray level k .

CDF of W is given as (Equation 6):

$$C_W(X_k) = \frac{1}{2}p_W(X_k) + \sum_{i=0}^{k-1} p_W(X_i), \quad (6)$$

For $k = 0, 1, \dots, L - 1$

The HE transformation over W is defined as (Equation 7):

$$T(X_k) = (L - 1)C_W(X_k), \quad (7)$$

For $k = 0, 1, \dots, L - 1$

2.6. Contrast Limited Adaptive Histogram Equalization (CLAHE)

To solve the problem of noise amplification by LHE, CLAHE was proposed [2]. In this method, the input image is decomposed into numerous non-overlapping regions with equal sizes. This division results in three different region groups: Corner Regions (CR), Border Regions (BR), and Inner Regions (IR) [19]. First, the histogram of each region is calculated. Then, to clip histograms based on the desired limit for contrast expansion, a limit is obtained and each histogram is redistributed considering the clip limit. In the end, CDFs of the resultant histograms are determined as transformation functions.

To limit the contrast to a desired level, the maximum slope of the histogram should be limited to a desirable value. To do so, the clip limit, β , is used which is given by the following Equation 8:

$$\beta = \frac{M}{L_R} \left(1 + \frac{\alpha}{100} (s_{max} - 1) \right) \quad (8)$$

Where M is the total number of pixels in one specific region and L_R is the total number of grayscales in that region. α is the clip factor and s_{max} is the maximum allowable slope. Note that by increasing α from zero to hundred, the maximum slope is increasing between one to s_{max} [19].

2.7. Adaptive Gamma Correction with Weighted Distribution (AGCWD)

Gamma correction is one of the HE techniques which uses a varying adaptive parameter γ . The Transform-based Gamma Correction (TGC) for gray level X_k is defined as (Equation 9):

$$T(X_k) = X_{max}(X_k/X_{max})^\gamma \quad (9)$$

Where X_{max} is the maximum intensity of the input image. Note that when $\gamma = 1$, the transformation curve of gamma correction is defined as identity curve and the gamma curves generated with $\gamma < 1$ have the opposite effect as gamma curves with $\gamma > 1$ [20].

Although the previous methods of HE enhance contrast and preserve brightness level of the input image, the resultant image is sometimes faced with over enhancement problem. Therefore, a new method was proposed as AGCWD. This method combines TGC and HE methods

and uses gamma to increase the brightness level of the low contrast image [7].

The AGCWD method utilizes CDF as an adaptive parameter to modify the transformation curve and maintains the statistical information of the histogram that might be lost by using other methods [20]. The proposed Adaptive Gamma Correction (AGC) is defined as (Equation 10):

$$T(X_k) = X_{max}(X_k/X_k)^{\gamma} = X_{max}(X_k/X_{max})^{1-C(X_k)} \quad (10)$$

In which $C(X_k)$ denotes the CDF at gray level X_k . The weighting process is applied by the normalized power law function in order to modify the histogram of the image according to gray levels frequency and avoid over enhancement [21]. The Weighting Distribution (WD) function is given in the following Equation 11:

$$p_w(X_k) = p_{max} \left(\frac{p(X_k) - p_{min}}{p_{max} - p_{min}} \right)^{\alpha} \quad (11)$$

Where α is the adjusting parameter, p_{min} and p_{max} are the minimum and maximum values of histogram PDF, respectively.

The modified CDF is defined as (Equation 12):

$$C_w(X_k) = \sum_{i=0}^k p_w(X_i) / \sum p_w \quad (12)$$

In which the sum of p_w is calculated as (Equation 13):

$$\sum p_w = \sum_{i=0}^{k_{max}} p_w(X_i) \quad (13)$$

In the end, the gamma parameter is modified as (Equation 14):

$$\gamma = 1 - C_w(i) \quad (14)$$

2.8. Gaussian Model-Based Histogram Equalization (GMHE)

One of the main problems of HE techniques discussed in previous sections (HE, BBHE, and DSIHE) is that they transform the background intensities toward brighter ones. This causes the image contrast to diminish. Another problem is that the mean or median of an infrared image seems not to be an appropriate indicator for decomposing the image and applying the equalization independently on both parts. Therefore, we propose to restrict the equalization just on the body region of the image. To do so, first, we need to eliminate the background from the image and then apply the equalization transform only on the body region.

According to the image histogram shown in Figure 1, one way to extract the body from the background is to use a threshold e.g., Otsu's method, and then apply the equalization between the threshold value and the maximum gray level. Here, we call this method threshold-based histogram equalization (TH/HE). As will be discussed in the result section, this approach can outperform BBHE and DSIHE based on some objective criteria but it still needs to get enhanced.

Another way to extract the body from the background can be achieved by considering the temperature information since background temperature is relatively low in comparison to body temperature. Gaussian distribution is one of the common distributions for modeling unimodal data [22]. Gaussian mixture model with K components is defined as (Equation 15, 16):

$$p(x) = \sum_{i=1}^K \phi_i N(x|\mu_i, \sigma_i) \quad (15)$$

$$N(x|\mu_i, \sigma_i) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left(-\frac{(x - \mu_i)^2}{2\sigma_i^2}\right) \quad (16)$$

Where N denotes the Gaussian distribution, ϕ_k is the mixture component weights, and μ_k and σ_k are the mean and variance of the k^{th} component, respectively. As Figure 1 depicts, the temperature histogram is unimodal data with two peaks. Thus, it is possible to fit a bimodal Gaussian density function to the temperature histogram. This procedure is shown in Figure 2. Due to the higher temperature of the body, the second Gaussian model is utilized. We assumed that the temperature range of the object (body) is between $m \pm 3\sigma$, where m is the mean and σ is the standard deviation of the second Gaussian model. As the scatter plot in Figure 1 shows, there is a linear relationship between temperature values and image gray levels. Therefore, we map the desired

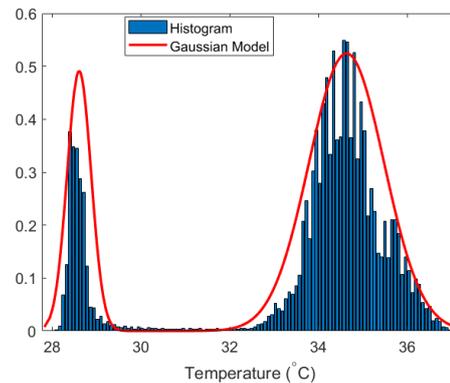


Figure 2. A bimodal Gaussian distribution fit on the temperature histogram

temperature range to the gray levels using the linear transformation parameters. In this way, the object and the background section in the image are separated which is shown in Figure 3.

Finally, we applied the HE to only the object section, which in this case is the body region. Here, we call this method the Gaussian model- based histogram equalization (GM/HE), and block diagram of the proposed method is shown in Figure 4.

3. Results

In this section, the performance of mentioned techniques is compared. Figure 5 shows a sample of the original image, its histogram, enhanced images by eight histogram equalizing methods, and the corresponding histograms. All of the methods were implemented on MATLAB version R2020b. For quantitative evaluation, four objective measures were used: AMBE, PSNR, SSI, and Entropy. In continue, these metrics are explained and obtained for all the methods as well as for every 50 images of the dataset.

3.1. Quantitative Evaluation based on the Absolute Mean Brightness Error

AMBE is used to measure brightness preservation in the processed images. AMBE for two images of X and the enhanced image (Y) is defined as (Equation 17):

$$AMBE(X, Y) = |\mu_X - \mu_Y| \quad (17)$$

Where μ_X and μ_Y are the mean of the input and the enhanced image, respectively.

A lower value of AMBE indicates better performance of the equalizing method in terms of preserving brightness and consequently, a better quality of the resultant image. Figure 6 shows the mean and the standard deviation of AMBE for all the 50 images and their enhanced versions obtained by the eight equalizing methods.

3.2. Quantitative Evaluation based on the Peak Signal-to-Noise Ratio

Assume that $X(i, j)$ is the input image with $M \times N$ pixels and $Y(i, j)$ is the enhanced image.

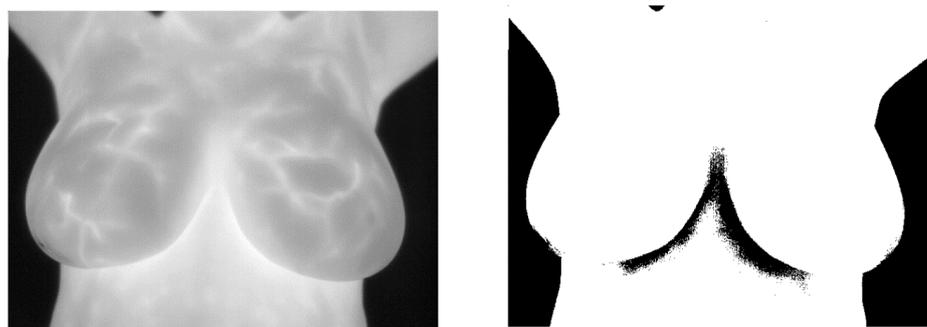


Figure 3. Left: Original image, right: Body region segmentation from the background by GM

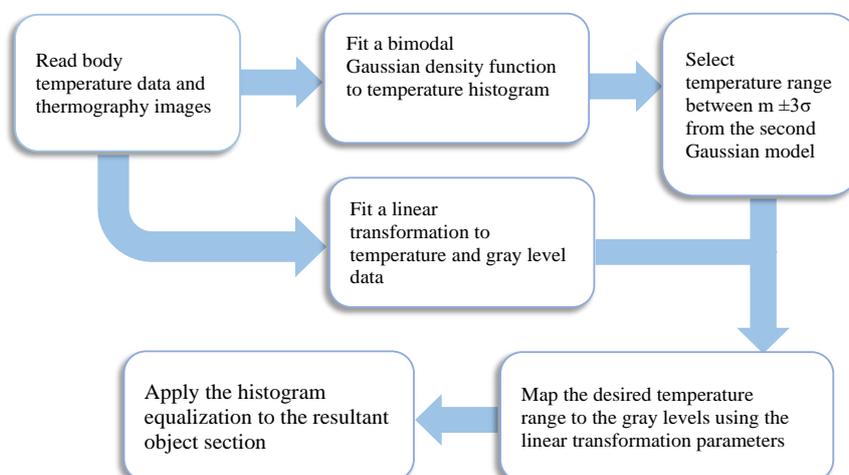


Figure 4. The block diagram of GM/HE method

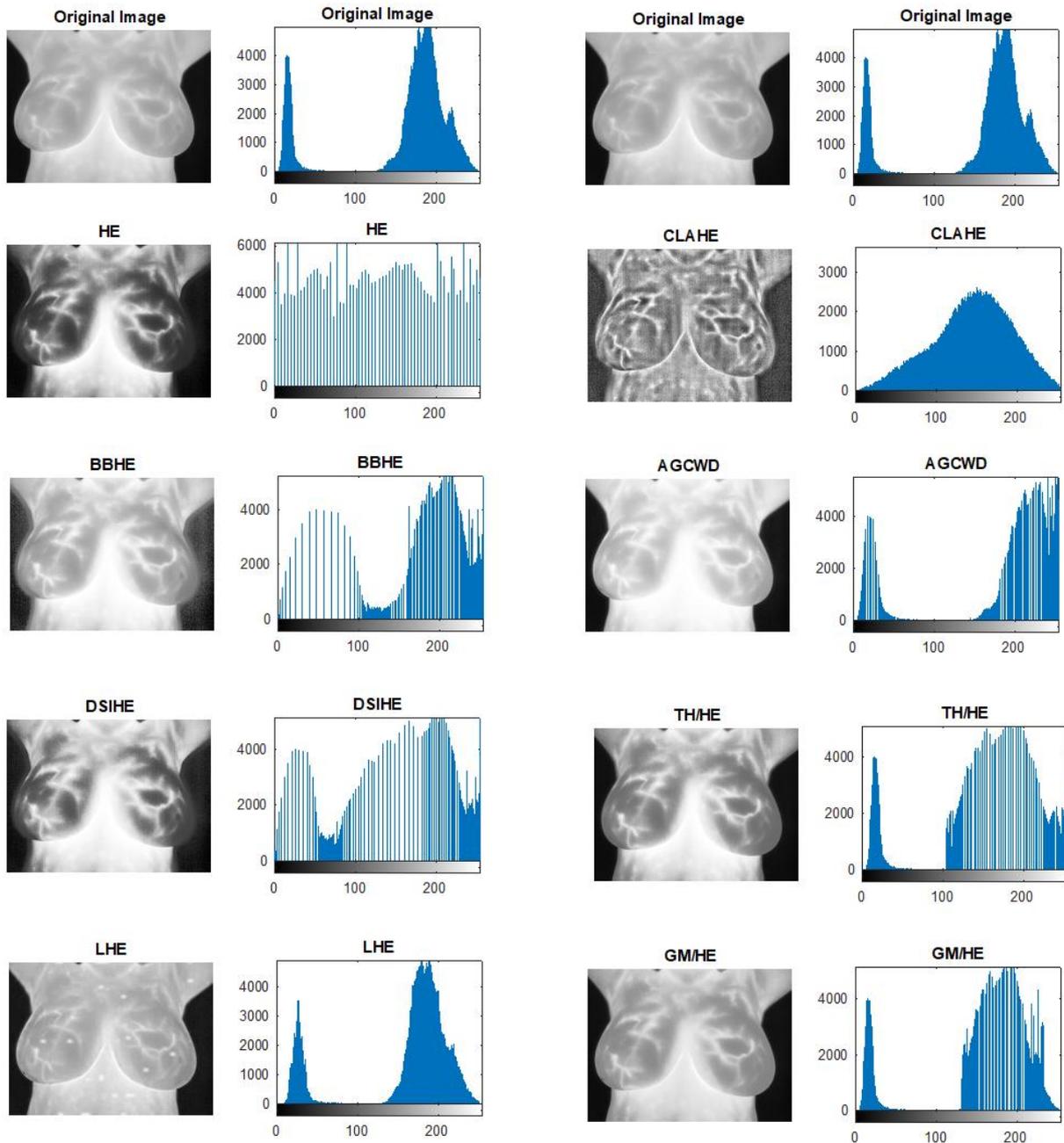


Figure 5. A sample of dataset image, the enhanced images by eight histogram equalization methods, and the correspondent histograms

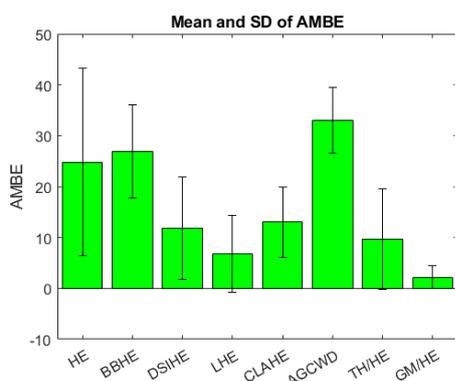


Figure 6. Absolute Mean Brightness Error (AMBE)

PSNR is calculated by (Equation 18):

$$PSNR = 10 \log_{10} \frac{(L - 1)^2}{MSE} \quad (18)$$

Where L is the maximum gray level and MSE is the mean squared error calculated as (Equation 19):

$$MSE = \frac{\sum_{i=1}^M \sum_{j=1}^N |X(i, j) - Y(i, j)|^2}{M \times N} \quad (19)$$

The higher value of PSNR means the better contrast enhancement. Figure 7 shows the mean and the standard

deviation of PSNR for all the 50 images and their enhanced versions obtained by the eight equalizing methods.

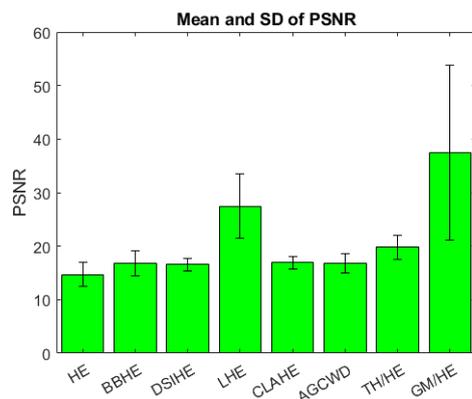


Figure 7. Peak Signal to Noise Ratio (PSNR)

3.3. Quantitative Evaluation based on the Structure Similarity Index

The SSI is a metric for measuring the similarity between two images.

For two images of X and Y , it is defined as (Equation 20):

$$SSI(X, Y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (20)$$

Where μ_x is the mean of the input image X , μ_y is the mean of the enhanced image Y , σ_x is the standard deviation of image X , σ_y is the standard deviation of image Y , σ_{XY} is the square root of covariance of image X and Y , and C_1 and C_2 are two constants.

This metric has a value between zero to one; a closer value to one illustrates the more similarity between input and the enhanced images. Figure 8 shows the mean and the standard deviation of SSI for all the 50 images and their enhanced versions obtained by the eight equalizing methods.

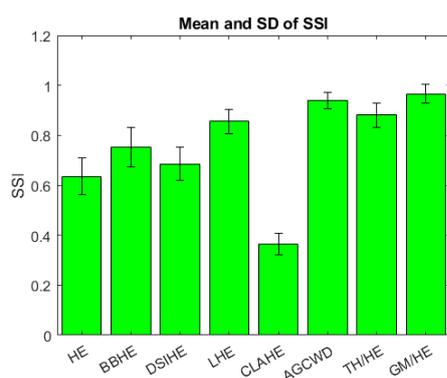


Figure 8. Structure Similarity Index (SSI)

3.4. Quantitative Evaluation based on the Entropy

The entropy of an image indicates the richness of the image details and is defined as (Equation 21):

$$Entropy[p] = - \sum_{k=0}^{L-1} p(X_k) \log_2 p(X_k) \quad (21)$$

Where p is the image PDF. Whatever the entropy of the enhanced image (E_Y) has a higher and closer value to the entropy of the original image (E_X), the intensity saturation effect, resultant by HE in higher intensities, decreases (Equation 22).

$$E_{diff} = E_X - E_Y \quad (22)$$

Figure 9 shows the mean and the standard deviation of entropy difference (E_{diff}) for all the 50 images and their enhanced versions obtained by the eight equalizing methods.

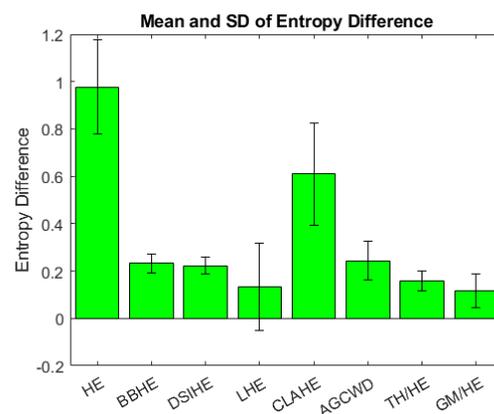


Figure 9. Entropy difference

4. Discussion

Thermography imaging is one of the effective and non-invasive methods in breast cancer screening. The acquired images are initially available as gray-level images. Increasing the image quality provides a better visual intuition and improves the specialist's diagnosis. One way of increasing the image quality is contrast enhancement that can be achieved by HE techniques. These techniques enhance the contrast of the image by redistributing the intensity of the gray levels in the whole dynamic range. A variety of methods based on HE is available some of which were reviewed in this paper. Among them are: GHE, BBHE, equal area DSIHE, LHE, CLAHE, and AGCWD.

Methods such as HE equalize the image globally meaning that it uses all of the gray levels in the histogram. This leads to transforming the background intensities toward the brighter ones and saturation may also occur meaning that the higher gray levels will be mapped to the maximum value. Consequently, this reduces the image contrast. On the other hand, the brightness of the image may not be preserved which is essential in medical images. Other methods such as BBHE, and DSIHE try to preserve the mean brightness by decomposing the image based on the mean and the median of the gray levels, respectively. But, as Figure 5 shows, mean and median may not be a proper indicator at least for breast thermography images. These methods also suffer from shifting the lower background intensities toward brighter ones causing contrast reduction. This problem also exists in LHE and CLAHE. These two methods also require to some parameters set manually such as window size and clip parameter and complicates the automatic preprocessing. Another important problem associated with LHE and CLAHE is spurious contours generated in the enhanced images that in fact means they manipulate the original image. Since thermography images are of medical type, therefore this problem cannot be overlooked. According to Figure 5, AGCWD solves the problem of lower gray level shifting but also suffers from saturation in higher gray levels.

To solve the mentioned problems, we proposed to apply the equalization method only on the object part of the image i.e., body. To do so, we proposed thresholding the gray level histogram by Otsu's method to first extract the body. Then, HE was applied to the second part of the histogram (TH/HE). This solves the problem of gray level shifting but not the saturation problem in higher intensities (Figure 5). To modify (TH/HE), we proposed to fit a bimodal Gaussian model on the temperature histogram rather than gray levels histogram. The first Gaussian models the low-temperature background and the second one models the body parts in the image (Figure 2). Mapping the temperatures in the range of $m \pm 3\sigma$ to the corresponding gray levels and restricting equalization on that range (GM/HE) not only solves the low gray level shift problem but also saturation as well as brightness preservation problem.

In this paper, we did not limit evaluation by just subjective visualization of the enhanced images. We also computed four quantitative metrics (Figures 6 to 9). As Figure 6 shows, based on AMBE criterion, GM/HE has the best and AGCWD has the worst performance. This means

the enhanced image by GM/HE has the least amount of manipulation and most resemblance to the original image, the two features that are important for medical image preprocessing. According to Figure 7, GM/HE has the highest PSNR although, the variance has also a high value. Checking the numerical values of PSNR for all the 50 images showed that the higher variance is attributed to a couple of images in the datasets. Based on SSI (Figure 8), CLAHE is the worst method and this is also apparent from Figure 5 since it produces artifactual contours. GM/HE has still better performance based on this metric. Finally, based on Figure 9, GM/HE has the least entropy difference among the methods.

Based on the results and explanations provided, we conclude that GM/HE is a HE method that considers the content of the medical image. Not only based on subjective visualization but also based on objective criteria, this method has a suitable performance in enhancing the quality of breast thermography images. Therefore, this method can help specialists to have better visualization and therefore an accurate diagnosis of breast abnormalities. In addition, as future work, we propose to use LHE in the framework of GM/HE since LHE seems to have better performance based on objective parameters.

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References

- 1- Yi-Sheng Sun *et al.*, "Risk factors and preventions of breast cancer." *International journal of biological sciences*, Vol. 13 (No. 11), p. 1387, (2017).
- 2- Anthony Muthu Arul Edwin Raj, Muniasamy Sundaram, and Thirassama Jaya, "Thermography based breast cancer detection using self-adaptive gray level histogram equalization color enhancement method." *International Journal of Imaging Systems and Technology*, Vol. 31 (No. 2), pp. 854-73, (2021).
- 3- Lulu Wang, "Early diagnosis of breast cancer." *Sensors*, Vol. 17 (No. 7), p. 1572, (2017).
- 4- Siva Teja Kakileti, Geetha Manjunath, Himanshu Madhu, and Hadonahalli Venkataramanappa Ramprakash, "Advances in breast thermography." in *IntechOpen*, p. 91, (2017).
- 5- Deepika Singh and Ashutosh Kumar Singh, "Role of image thermography in early breast cancer detection-Past,

- present and future." *Computer methods and programs in biomedicine*, Vol. 183p. 105074, (2020).
- 6- Deborah A Kennedy, Tanya Lee, and Dugald Seely, "A comparative review of thermography as a breast cancer screening technique." *Integrative cancer therapies*, Vol. 8 (No. 1), pp. 9-16, (2009).
- 7- Monika Agarwal and Rashima Mahajan, "Medical image contrast enhancement using range limited weighted histogram equalization." *Procedia Computer Science*, Vol. 125pp. 149-56, (2018).
- 8- R Sumathi, M Venkatesulu, and Sridhar P Arjunan, "Segmenting and Classifying MRI Multimodal Images Using Cuckoo Search Optimization and KNN Classifier." *IETE Journal of Research*, pp. 1-8, (2021).
- 9- Haichao Cao, Shiliang Pu, Wenming Tan, and Junyan Tong, "Breast mass detection in digital mammography based on anchor-free architecture." *Computer Methods and Programs in Biomedicine*, Vol. 205p. 106033, (2021).
- 10- Jyoti Dabass and Manju Dabass, "Denoising, Edge Correction, and Enhancement of Breast Cancer Ultrasound Images." in *Advances in Communication and Computational Technology: Springer*, (2021), pp. 1153-72.
- 11- Saeed Kermani, Nasser Samadzadehaghdam, and Mahnaz EtehadTavakol, "Automatic color segmentation of breast infrared images using a Gaussian mixture model." *Optik*, Vol. 126 (No. 21), pp. 3288-94, (2015).
- 12- Nicholas A Diakides and Joseph D Bronzino, *Medical infrared imaging*. CRC press, (2007).
- 13- EFJ Ring and Kurt Ammer, "The technique of infrared imaging in medicine." *Thermology international*, Vol. 10 (No. 1), pp. 7-14, (2000).
- 14- Omprakash Patel, Yogendra PS Maravi, and Sanjeev Sharma, "A comparative study of histogram equalization based image enhancement techniques for brightness preservation and contrast enhancement." *arXiv preprint arXiv:1311.4033*, (2013).
- 15- Nicholas Sia Pik Kong, Haidi Ibrahim, and Seng Chun Hoo, "A literature review on histogram equalization and its variations for digital image enhancement." *International Journal of Innovation, Management and Technology*, Vol. 4 (No. 4), p. 386, (2013).
- 16- Yu Wang, Qian Chen, and Baeomin Zhang, "Image enhancement based on equal area dualistic sub-image histogram equalization method." *IEEE Transactions on consumer Electronics*, Vol. 45 (No. 1), pp. 68-75, (1999).
- 17- Yeong-Taeg Kim, "Contrast enhancement using brightness preserving bi-histogram equalization." *IEEE Transactions on consumer Electronics*, Vol. 43 (No. 1), pp. 1-8, (1997).
- 18- R Dale-Jones and Tardi Tjahjadi, "A study and modification of the local histogram equalization algorithm." *Pattern Recognition*, Vol. 26 (No. 9), pp. 1373-81, (1993).
- 19- Ali M Reza, "Realization of the contrast limited adaptive histogram equalization (CLAHE) for real-time image enhancement." *Journal of VLSI signal processing systems for signal, image and video technology*, Vol. 38 (No. 1), pp. 35-44, (2004).
- 20- Shih-Chia Huang, Fan-Chieh Cheng, and Yi-Sheng Chiu, "Efficient contrast enhancement using adaptive gamma correction with weighting distribution." *IEEE transactions on image processing*, Vol. 22 (No. 3), pp. 1032-41, (2012).
- 21- Mary Kim and Min Gyo Chung, "Recursively separated and weighted histogram equalization for brightness preservation and contrast enhancement." *IEEE Transactions on consumer Electronics*, Vol. 54 (No. 3), pp. 1389-97, (2008).
- 22- Bruce G Lindsay, "Mixture models: theory, geometry and applications." in *NSF-CBMS regional conference series in probability and statistics: JSTOR*, pp. i-163, (1995).